

Lab #2: Fluid Properties, Conservation of Mass, & Open Channel Flow Laboratory

Objectives: The objectives of this laboratory are to measure the density of water, check fluid viscosity through a Reynolds number criterion, and estimate the terms in the conservation of water mass equation.

PART I

Introduction – Fluid Properties and Conservation of Mass

Density

$$\rho = m/V \quad (1)$$

where, ρ is the density, m is the mass of water, and V is the volume of the water. In general, density can be changed by changing either the pressure or the temperature. Increasing the pressure will always increase the density of a material. Increasing the temperature generally decreases the density, but there are notable exceptions to this generalization. For example, the density of water increases between its melting point at 0°C and 4°C; That is why it is important to record the temperature of the water for the density experiment.

Reynolds criterion

The Reynolds number gives a measure of the ratio of inertial forces ($\rho V^2/L$) to viscous forces ($\mu V/L^2$). It is dimensionless, where D is the hydraulic diameter (can be calculated by cross section of short flexible tubing section), v is the mean fluid velocity (can be calculated using discharge and tube cross section), ν is kinematic viscosity (read from White, Fluid Mechanics, McGraw Hill, 2008).

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho V^2/L}{\mu V/L^2} = \frac{VD}{\nu} \quad (2)$$

Laminar if $Re < 2300$
 Transition if $2300 < Re < 4000$
 Turbulent $Re > 4000$

Table 1: Kinematic Viscosity

T (°C)	(m ² /s) x 10 ⁻⁶
0	1.7875
5	1.519
10	1.307
20	1.004
30	0.801
40	0.658
50	0.553
60	0.475
70	0.413
80	0.365
90	0.326
100	0.294

Conservation of mass equation for a constant density fluid

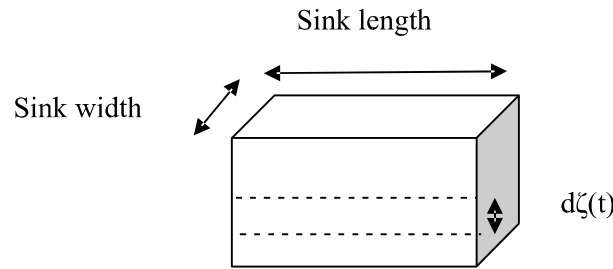


Figure 1. Sketch of the sink.

In any steady state process, the rate at which mass enters a system is equal to the rate at which mass leaves the system. For this experiment, the mass of water in the sink at the beginning should equal to the total mass of water leaves the sink after it drained.

$$\rho \cdot Q_{in}(t) \cdot t_{fill} = \rho \cdot \int_0^t Q_{out}(t) dt = \rho \cdot \int_0^t A \cdot \frac{d\zeta(t)}{dt} dt \quad (3)$$

Materials

(Maintained in Marston 10):

- 10 mL, 250 mL, and 4L graduated cylinders
- Balance
- Flexible tubing sections
- Sink with stopper
- Stop watch and scale
- Thermometer

Procedure

Density Experiment and Slow Jet Experiment

Density Experiment;

1. *Measure the weight of the water:*
 - a. Weigh the graduated cylinder when empty.
 - b. Then, fill the graduated cylinder to the 250 mL mark and weigh again.
 - c. Repeat the measurements three times, recording the weights each time on data sheet.

Slow Jet Experiment;

2. *Measure the internal diameter of the tubing through the following procedure (each group will have one piece of flexible tubing which will be used consistently throughout the experiment):*
 - a. Measure the length of the tube.
 - b. Fill the flexible tubing section with some water by immersing it in a full graduated cylinder, placing your finger on the top and removing the tube (now full) from the cylinder.
 - c. Pour the water from the tube into an empty graduated cylinder. Measure the volume of water in the tube.
 - d. Note the water temperature

- e. You can now calculate the internal diameter based on the volume measured, the length of the column of water, and the equation for volume of a cylinder.

Note: Volume of a cylinder

$$V = \pi r^2 h \quad (4)$$

3. *Measure the flow rate of the slow jet:*

- a. Attach the tubing section to the sink tap and turn on the faucet to a slow flow. Hold the tube up so that a free jet exits into the sink. This jet should be smooth and clear as it hits the sink.
- b. Use the stopwatch to measure the time required to fill the 250 mL graduated cylinder.
- c. Repeat twice and record the measurements.
- d. Turn off the faucet and disconnect the tubing.

Fast Jet Experiment and Conservation of Mass

4. *Measure the flow rate of the fast jet:*

- a. Note the water temperature.
- b. Attach the long flexible tubing section to the sink tap and turn on the faucet to a fast flow. Hold the tube up so that an (approximately) 0.5 m long free jet exits into the sink. This jet should become irregular before it hits the sink.
- c. Use the stopwatch to measure the time required to fill the 4 L cylinder.
- d. Turn off the faucet.

Conservation of Mass Experiment;

5. *Measure the discharge:*

- a. Measure the horizontal cross-sectional area of the sink.
- b. Turn on the faucet and measure the discharge by observing the time required to fill the sink.
- c. Put a stopper into the sink and measure the time required for the free surface in the sink to rise 17 cm, using the stopwatch and scale.
- d. Turn off the faucet and disconnect the tubing.
- e. Now pull out the stopper and observe the time required to drop each centimeter until the sink fully drains, using the scale and the stopwatch. You should have 17 times recorded, with time accumulating.

Lab Report Part I Components

Methods section (Theory):

- State the definition of water density, ρ .
- Show how the filled tube and the volume data yield an estimate for the average tube velocity, v .
- State the Reynolds' number criterion for laminar (slow) and turbulent (fast) flow.
- Express the conservation of mass equation for a constant density fluid in terms of volumetric discharges, Q_{in} , Q_{out} , and water levels, $\zeta(t)$, in the sink. This should be a first order differential equation, with time as the independent variable. Be sure to include an initial condition.
- Use a manual sketch of the sink to define flow variables and to identify appropriate control surfaces and the control volume.

Data Analysis:

- Compute the individual water density estimates from your volume and mass data.
- Compute the average Reynolds number for the slow and fast flow experiments.
- Use the data to evaluate the constant volumetric discharges
- Use the data to evaluate the storage changed, ζ/dt , when the sink is filling from the time taken to fill the sink and the measured horizontal cross section area of the sink.
- Use the time increments between the falling sink levels and the conservation of mass equation to estimate the variable drainage discharge.

Results and Discussion:

- How do your observed densities compare with the literature values, and how variable are your densities?
- How does your Reynolds number criterion for laminar and turbulent flow compare with the literature value of 4000?
- Compare Q_{in} from the filling of the cylinder with the Q_{in} from the observed storage change during sink filling, and comment.
- Plot Q_{out} and ζ versus time (as symbols) during sink drainage. Is Q_{out} constant?
- Suppose that we have for the sink drainage, with the drainage coefficient C_{out} :

$$Q_{out} = C_{out}\sqrt{\zeta} \quad (5)$$

Derive a mathematical expression for t . Estimate C_{out} (show units) by plotting a curve through the data.

**Data Sheet
 Part I**

Density Experiment

Trial	Volume (mL)	Empty Cylinder (g)	Full Cylinder (g)	Density (g/cm ³)	Density (kg/m ³)
1	250				
2	250				
3	250				

Notes:

Slow Jet Experiment

Tube Characteristics

Water Temperature (°C)	
Tube Volume (mL)	
Tube Length (cm)	
Internal Diameter (cm)	
Area (cm ²)	
Kinematic Viscosity (m ² /s)	

	Volume (mL)	Fill Time (s)	Flow Rate (cm ³ /s)	Velocity (cm/s)	Velocity (m/s)	Reynolds Number
Trial 1	250					
Trial 2	250					

Notes:

Fast Jet Experiment

Tube Characteristics

Water Temperature (°C)	
Tube Volume (mL)	
Tube Length (cm)	
Internal Diameter (cm)	
Area (cm ²)	
Kinematic Viscosity (m ² /s)	

	Volume (mL)	Fill Time (s)	Flow Rate (cm ³ /s)	Velocity (cm/s)	Velocity (m/s)	Reynolds Number
Trial 1	4000					
Trial 2	4000					

Notes:

Conservation of Mass Experiment (Filling)

Sink width, cm: _____ Sink Length, cm: _____ Sink area, cm²: _____

Sink depth, cm: 17cm Sink Volume, cm³: _____

Fill time, sec: _____ Q_{in} (jet), cm³/s: _____ Q_{in} (mass), kg/s: _____

Notes:

Conservation of Mass Experiment (Draining the sink)

Elevation ζ , cm	Time t, sec	Δt sec	$\Delta\zeta/\Delta t$ cm/sec	Q_{out} cm^3/s
17	0	N/A	N/A	N/A
16				
15				
14				
13				
12				
11				
10				
9				
8				
7				
6				
5				
4				
3				
2				
1				

Notes:

PART II Introduction – Open Channel Flow

Open-channel flow is a type of fluid flow that is not entirely included within rigid boundaries; a part of the flow is in contact with empty space (Figure 1) (Chanson, 2004). The open channel flows are driven by gravity alone, and the pressure gradient at the atmospheric interface is negligible. The surface of the flow thus formed is called a **free surface**, because that flow boundary is freely deformable, in contrast to the solid boundaries. The basic force balance in an open channel is between gravity and friction. The other type of flow within a conduit is **pipe flow**.

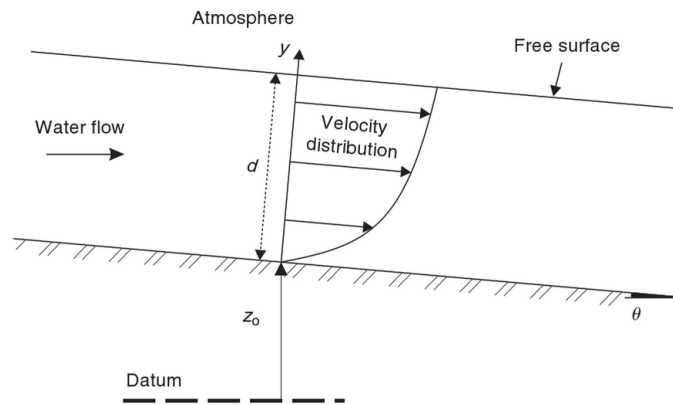


Figure 2. Sketch of open channel flow

Reynold's number of open channel flow

As in pipe flow, three states of flow are possible, laminar, turbulent and transitional. The effects of viscosity relative to inertial forces of the flow determine the state of flow. The relative effects of velocity to inertia can be represented by the Reynold's number which is a dimensionless number derived through dimensional analysis of the flow. The Reynold's number for open channel flow is given by:

$$Re = \frac{VR_h}{\nu} \quad (1)$$

Where, Re is the Reynold's number; V is the mean flow velocity; R_h is a characteristic length (the characteristic length for open channel flow is the hydraulic radius) and ν is the kinematic viscosity of the fluid. R_h is calculated using the cross-sectional area (A) divided by wetted perimeter (P) (Equation 2), where b is the width of the channel, and y is the depth of water in the channel. For the full pipe, hydraulic radius is calculated by Equation 3 (Figure 2).

$$R_h = \frac{A}{P} ; P = b + 2y \quad (2)$$

$$\text{Full pipe: } R_h = \frac{1}{4} D_h \quad (3)$$

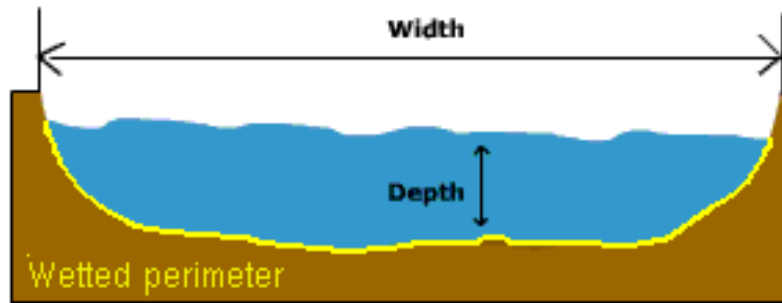


Figure 3. graphic description of wetted perimeter

Part A: V-notch weir in open channel flow

Weirs are overflow structures built across artificial open flow channels to measure the volumetric rate of water flow (flowrate) (Figure 3). Some general terms pertaining to weirs are:

- Notch: the opening which water flows through
- Crest: the edge which water flows over
- Nappe: the overflowing sheet of water
- Length: the “width” of the weir notch

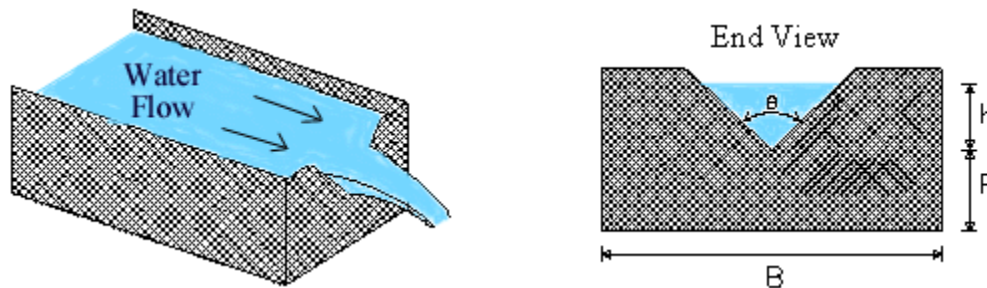


Figure 4. Sketch of V-notch weir in open channel

The basic principle is that discharge is directly related to the water depth above the crotch (bottom) of the V; this distance is called head (h). The V-notch design causes small changes in discharge to have a large change in depth allowing more accurate head measurement than with a rectangular weir.

Equation 4, 5, and 6 are used to calculate the liquid flowrate (Figure 3). h_1 is the head associated with velocity at a point in the weir; h_2 is head associated with the total height of the weir. g is gravitational constant. b is the width of free surface, and α is the angle of the V-notch weir, which are used to calculate the cross-section area. Equation 4 is the presentation of Bernoulli equation when the surface pressure is zero. Q is fluid discharge (flowrate): at any given point, the flowrate is the product of fluid velocity (V) and cross-section area (A) (Eq. 5). However, the flowrate changes when hydraulic head changes (Eq. 7).

$$h_1 = \frac{V^2}{2g} \tag{4}$$

$$Q = VA; A = bh \tag{5}$$

$$b = 2(h_2 - h_1) \tan \frac{\alpha}{2} \tag{6}$$

$$\int dQ = \int (\sqrt{2gh_1}) [2(h_2 - h_1) \tan \frac{\alpha}{2}] dh_1 \tag{7}$$

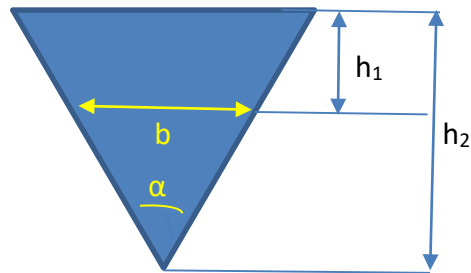


Figure 5. sketch of discharge calculation

The result of integration is:

$$Q = C_D \sqrt{g} \tan\left(\frac{\alpha}{2}\right) H^{\frac{5}{2}} \quad (8a)$$

$$C_D = \frac{8\sqrt{2}}{15} \quad (\text{Theoretical}) \quad (8b)$$

with gravitational acceleration g , head H above the base of the notch, and C_D is discharge coefficient. The theoretical value represented by Eq. (8b) precludes losses, upstream kinetic energy, and jet contraction.

Reference:

Chanson, H. (2004) *The hydraulics of open channel flow: an introduction; basic principles, sediment motion, hydraulic modelling, design of hydraulic structures*, Amsterdam; Boston: *Elsevier Butterworth Heineman*

Procedure:

Apparatus:

The apparatus consists of the following equipment, manufactured by G. Cussons, Limited, of Great Britain, and maintained in Marston 10:

- Hydraulic bench (P. 6100)
- Hook gauge and scale (P. 6107)
- Rectangular open channel (P. 6245)
- 90 degree (α) V-notch weir
- Stop watch and scale

V-Notch Weir

1. Note the water temperature in the hydraulic bench.
2. Note the cross-sectional area of the drainage canal approaching the weir.
3. Measure the height of the bottom of the “V” to the bottom of the weir, as well as the cross-sectional area that will be used to measure the volume of flow (referring to the low and high flow regions in the stepped measuring tank of the hydraulic bench).
4. Establish a low recirculating flow in the bench by turning the pump on and maintaining a low setting on the rheostat, with the measuring tank drain open. The discharge should be sufficient to create a free jet through the V-notch weir. Observe the progression of flow through the drainage canal, V-notch weir, and into the measuring tank and sump.
5. Record the water surface elevation in the drainage canal.
6. Be ready with a stopwatch. When measuring the flow rate, select a volume in the lower step of the tank (as well as the high step if chosen to do so). Turn the knob at the sump horizontal to stop drainage and allow for flow to accumulate to the selected volume. The timer should start at the time the drainage of the sump is stopped. There should be a total of 2 trials for this flow.
7. Make replicate measurements for each new flow, so that you'll have a total of 2 trial flows for 3 different flows (giving a total of 6 data points) for the V-notch weir experiment.
8. Turn off the pump and open the measuring tank drain.

Data Analysis:

V-notch weir

Estimate observed values for C_D and the open channel Reynolds number (defined in the instructional lab, based on the upstream section) for the 3 discharges of the V-notch weir. Neat, handwritten calculations are appropriate, with inclusion of a control volume sketch. State all assumptions.

Notes for the Results and Discussion section:

- Why would you expect C_D observed to be less than the theoretical Eq. (1b)? Does C_D vary with Reynolds number?

Data Sheet

V-Notch Weir

Temp (C): _____ Approach width, cm: _____

Height of the bottom of the “V” to the bottom of the weir, cm: _____

Area high Q section cm²: _____ Area low Q, cm²: _____

Low Flow

Trial	$\Delta\xi$ (cm)	Δt (s)	Q (cm ³ /s)	h ₁ (cm)	h ₂ (cm)	C _D
1						
2						

Mid Flow

Trial	$\Delta\xi$ (cm)	Δt (s)	Q (cm ³ /s)	h ₁ (cm)	h ₂ (cm)	C _D
1						
2						

High Flow

Trial	$\Delta\xi$ (cm)	Δt (s)	Q (cm ³ /s)	h ₁ (cm)	h ₂ (cm)	C _D
1						
2						